

# The Recoverability of P-technique Factor Analysis

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**DOI:** 10.1080/00273170802620204

**Publication Frequency:** 6 issues per year

**Published in:**  **Multivariate Behavioral Research**, Volume 44, Issue 1 January 2009 , pages 130 - 141

## Abstract

It seems that just when we are about to lay P-technique factor analysis finally to rest as obsolete because of newer, more sophisticated multivariate time-series models using latent variables—dynamic factor models—it rears its head to inform us that an obituary may be premature. We present the results of some simulations demonstrating that even though it does not explicitly model lagged information, P-technique's ability to recover the parameters of underlying dynamic processes involving lagged relations among the manifest variables is apparently robust and accurate. An empirical example is presented using 103 days of affective mood self-ratings from a young pregnant woman. Implications of the simulation and empirical findings are briefly discussed.



The systematic, multivariate study of intraindividual variability is receiving more and more attention. A promising line of research in this regard is focused on individual level analysis with aggregation and synthesis coming only after individual level data have been examined and findings used to inform subsequent aggregation steps (see, e.g., Molenaar, 2004; Nesselroade & Ford, 1985; Zevon & Tellegen, 1982). Many of the gains made in studying intraindividual variability have come about because of the growing availability of analytical tools for modeling multivariate time-series data.

One of the defining methodological contributions to individual level study—P-technique factor analysis—was made by Cattell, Cattell, & Rhymer, 1947, and Cattell, 1963, beginning well over a half-century ago. P-technique factor analysis involves the application of the common factor model to multivariate, repeated measurements of one individual obtained over a substantial number of occasions. The findings from a series of P-technique analyses helped to define and sharpen the study of intraindividual variability in several ways, including the trait-state distinction in personality and clinical research (e.g., Cattell & Scheler, 1961). For reviews of this literature see Luborsky and Mintz (1972) and Jones and Nesselroade (1990).

Within the context of multivariate investigation, several authors have argued for the value of individual-level analysis outcomes as a basis for conducting more informed aggregation of information over multiple individuals (Lebo &

Nesselroade, 1978; Molenaar, 2004; Molenaar, Huizenga, & Nesselroade, 2003; Nesselroade & Ford, 1985; Zevon & Tellegen, 1982). Despite the early promise of the P-technique approach for capturing and representing behavior at the individual level, it was recognized as being flawed by several writers, including Cattell himself (Cattell, 1963) because the technique did not take into account lag information in the multivariate time-series. A series of important discussions further explicated the problems and presented various solutions for dealing with them (Anderson, 1963; Brown & Nesselroade, 2005; Holtzman, 1963; Molenaar, 1985). Over the past 25 years or so a class of procedures generally referred to as *dynamic factor analysis* (DFA) has been presented and used by various investigators. DFA explicitly models lags in the relations between the factors and observed variables either directly or indirectly, depending on the model being fitted (see, e.g., Brown & Nesselroade, 2005; Nesselroade, McArdle, Aggen, & Meyers, 2002).

Despite the criticisms directed at it and the promise of the newer DFA methods, the lure of P-technique factor analysis has been remarkably robust over many decades. Part of the attractiveness, we believe, is the combination of P-technique's relative simplicity and the fact that one of the more intuitively appealing paths to apprehending process is to obtain many repeated measurements of individuals. And indeed P-technique factor analysis was promoted by Cattell (1963) for the study of within-individual traits or processes—even though the model did not provide for the representation of any kind of temporal continuity such as lagged relations.

So, the questions are, How and how much does P-technique factor analysis distort the character of one's data when it is applied to multivariate time-series containing lagged relations? Does it supply its somewhat limited representation of the data in a robust and reliable portrayal? We narrowed these questions to that of how well does P-technique estimate the parameters and process information of a state-space representation compared with more advanced state-space modeling procedures? We then performed some simulations pitting traditional P-technique factor analysis against a state-space representation that included one-lag relations to evaluate their relative ability to recover parametric information from data known to harbor lagged relations. We also conducted a comparative analysis using empirical data consisting of daily affect self-reports of a young, pregnant female participant spanning 103 successive days.

## THE SIMULATION

The history of critiques of P-technique and we believe the weight of current thinking on these matters caution against the adequacy of P-technique factor analysis as a tool for modeling any aspects of multivariate time-series data. This simulation study was designed to compare classical P-technique with state-space modeling procedures in terms of the former's ability to recover both the parameter values and the latent state process information in auto- and lagged cross-correlated multivariate time-series that were generated from an underlying state-space model.

The simulation was based on the following state-space model comprising five indicators and two factors:

$$y_1(t) = 1.0\eta_1(t) + 0.0\eta_2(t) + \epsilon_1(t)$$

$$y_2(t) = 0.9\eta_1(t) + 0.7\eta_2(t) + \epsilon_2(t)$$

$$y_3(t) = 0.8\eta_1(t) + 0.8\eta_2(t) + \epsilon_3(t)$$

$$y_4(t) = 0.7\eta_1(t) + 0.9\eta_2(t) + \epsilon_4(t)$$

$$y_5(t) = 0.0\eta_1(t) + 1.0\eta_2(t) + \epsilon_5(t).$$

The measurement error series  $\epsilon_j(t)$ ,  $j = 1, 2, 3, 4, 5$ , all are Gaussian white noise, mutually uncorrelated, having no sequential dependency, means equal to zero, and variances equal to 0.5. Other characteristics of the simulated data are

$$\eta_1(t) = 0.8\eta_1(t-1) + a_1(t) \text{ and}$$

$$\eta_2(t) = 0.3\eta_1(t-1) + 0.5\eta_2(t-1) + a_2(t).$$

The Gaussian innovation series  $a_1(t)$  and  $a_2(t)$  are mutually uncorrelated, have mean zero, and variance 1.0.

Thus, the bivariate  $\eta$  state-process has high autocorrelation and lagged cross-correlation. Hence, the observed five-variate series is likewise highly auto- and lagged cross-correlated; all the more so because  $y_j(t)$ ,  $j = 2,3,4$ , are mixtures of the latent bivariate  $\eta$  process.

Two simulation conditions varying the length of the time-series ( $T$ ) were used. The two conditions were  $T = 50$  and  $T = 300$  observations. These two values were chosen because 50 represents a "comfortable" number of observations for those who know that time-series need to be respectably long but resist "rules of thumb" such as "at least 100 observations" and are conversant with the realities of collecting frequently repeated measurements. Three hundred was chosen because it seems to be respectable from a time-series modeler's perspective. Our private sample of behavioral scientists could not be expected to go higher than 300; keeping them above 49 is likely a more justifiable concern. For both of these simulation conditions, we ran 500 replications.

Summarizing in matrix notation, the simulation model is

$$y(t) = \Lambda \eta(t) + \epsilon(t) \quad (1)$$

$$\eta(t) = B \eta(t-1) + \zeta(t) \quad (2)$$

where

$$\text{where } \Lambda = \begin{bmatrix} 1.0 & 0.0 \\ 0.9 & 0.7 \\ 0.8 & 0.8 \\ 0.7 & 0.9 \\ 0.0 & 1.0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.8 & 0.0 \\ 0.3 & 0.5 \end{bmatrix}.$$

$$\text{Also, } \text{cov}[\epsilon(t), \epsilon(t-u)'] = \begin{bmatrix} 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.5 \end{bmatrix}$$

if  $u = 0$

=  $\mathbf{0}$  (the null matrix) if  $u \neq 0$  and

$$\text{cov}[\zeta(t), \zeta(t-u)'] = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

if  $u = 0$

if  $u \neq 0 = \mathbf{0}$  if  $u \neq 0$ . Thus, all factor loadings are free save for  $\Lambda(1,1)$ ,  $\Lambda(5,1)$ ,  $\Lambda(1,2)$ , and  $\Lambda(5,2)$ . The loadings  $\Lambda(1,1)$  and  $\Lambda(5,2)$  are fixed at 1.0 in order to scale  $\eta_1$  and  $\eta_2$ . Only the loadings  $\Lambda(5,1)$  and  $\Lambda(1,2)$ , both fixed at 0.0, are genuinely

fixed loadings. The variances and the covariance of  $\eta_1$  and  $\eta_2$  are free. Also all measurement error variances are free. The P-technique factor model being fitted is simply an oblique factor model given by Equation (1), whereas the state space model given by Equations (1) and (2) constitutes a specific instance of the dynamic factor model in which there are no lagged factor loadings.

To fit the state-space model for each simulated series, a  $10 \times 10$  block-Toeplitz covariance matrix was estimated following the procedures of Molenaar (1985). Each of the two symmetric portions of the block-Toeplitz matrix contained the zero lag  $5 \times 5$  covariance matrix and the asymmetric portion contained the lag-one  $5 \times 5$  covariance matrix. The state-space model was fitted to it by means of Nonlis (a structural equation modeling program that permits arbitrary linear and nonlinear equality and inequality constraints; Dolan & Molenaar, 1994). The bivariate state process was estimated by means of the Kalman filter.

The classical 2-factor P-technique model was fitted to the  $5 \times 5$  zero lag covariance matrix of the observed series. This covariance matrix is the same as the upper symmetric portion of the block-Toeplitz matrix described earlier. The P-technique model was also fitted using Nonlis. The bivariate state process (factor scores over occasions) was estimated by means of the regression estimator for factor scores.

### First Simulation Condition— $T = 50$

In the first condition, five-variate  $y(t)$  series were generated according to the model summarized earlier for  $t = 1, \dots, 50$ . The parameter estimates (loadings) averaged across the 500 replications for the state-space modeling and P-technique modeling are presented in the upper portion of Table 1. The corresponding standard deviations of the 500 replications are given in parentheses. In the upper portion of Table 2, the average correlations between the true process scores,  $\eta(t)$ , and estimates of these true process scores are presented for the state-space and P-technique modeling approaches. As noted earlier, to estimate the process scores, a Kalman filtered estimate was used for the state-space modeling and a regression estimate was used in the case of the P-technique modeling.

**TABLE 1 True Factor Loadings and Those Estimated via State-Space and Conventional P-technique Modeling (Standard Deviations in Parentheses)**

| Variable | Estimated Loading |          |                   |             |                   |             |
|----------|-------------------|----------|-------------------|-------------|-------------------|-------------|
|          | True Loading      |          | State-Space Model |             | P-technique Model |             |
|          | Factor 1          | Factor 2 | Factor 1          | Factor 2    | Factor 1          | Factor 2    |
| 1        | 1.000             | .000     | 1.000             | .000        | 1.000             | .000        |
| 2        | .900              | .700     | .919 (.211)       | .697 (.222) | .918 (.235)       | .695 (.242) |
| 3        | .800              | .800     | .823 (.191)       | .793 (.216) | .827 (.249)       | .784 (.319) |
| 4        | .700              | .900     | .710 (.198)       | .898 (.229) | .708 (.228)       | .899 (.267) |
| 5        | .000              | 1.000    | .000              | 1.000       | .000              | 1.000       |
| 1        | 1.000             | .000     | 1.000             | .000        | 1.000             | .000        |
| 2        | .900              | .700     | .906 (.065)       | .697 (.071) | .909 (.119)       | .694 (.133) |
| 3        | .800              | .800     | .804 (.066)       | .798 (.074) | .808 (.118)       | .793 (.137) |
| 4        | .700              | .900     | .704 (.067)       | .894 (.078) | .710 (.120)       | .888 (.144) |
| 5        | .000              | 1.000    | .000              | 1.000       | .000              | 1.000       |

*Note.* All estimates are based on 500 replications. Time-series are 50 occasions in length (upper portion of table) and 300 occasions in length (lower portion of table).

**TABLE 2 Averaged Correlations Between True Process  $\eta(t)$  and Estimates of  $\eta(t)$ . Averages**

### **Are Over 500 Replications (Standard Deviations are in Parentheses)**

| Variables Correlated   | Factor 1    | Factor 2    |
|--|-------------|-------------|
| $\bar{r}$ [true $\eta(t)$ - Kalman filtered $\eta(t)$ ]      | .933 (.035) | .917 (.046) |
| $\bar{r}$ [true $\eta(t)$ - regression estimated $\eta(t)$ ] | .923 (.035) | .906 (.040) |
| $\bar{r}$ [true $\eta(t)$ - Kalman filtered $\eta(t)$ ]      | .957 (.009) | .944 (.009) |
| $\bar{r}$ [true $\eta(t)$ - regression estimated $\eta(t)$ ] | .949 (.014) | .934 (.016) |

*Note.* Time-Series are 50 Occasions in Length (Upper Portion of Table) and 300 Occasions in Length (Lower Portion of Table).

### **Second Simulation Condition—T = 300**

The second simulation was conducted exactly as the first except that the time-series involved 300 occasions instead of 50. The estimated loadings are presented in the lower portion of Table 1. The correlations between the true process factor scores and those obtained by the two estimation procedures are given in the lower portion of Table 2.

The general outcome pattern across both levels of the simulation can be summarized as follows: First, the parameter estimates obtained by means of classical P-technique closely correspond to those obtained by state-space modeling (according to the approach detailed by Molenaar, 1985). Second, the standard deviations are slightly larger for P-technique parameter estimates than for state-space modeling parameter estimates in the  $T = 50$  condition. In the  $T = 300$  condition, however, the standard deviations for P-technique parameter estimates are about twice as large as the standard deviations for state-space modeling parameter estimates. Third, the correlations between the Kalman filtered bivariate state process and the true state process (used in the simulations) are very high in both the  $T = 50$  and  $T = 300$  conditions and have small standard deviations. Fourth, the correlations between the regression estimate of the state process obtained in classical P-technique and the true state process (used in the simulations) are almost as high as those obtained with Kalman filtering in state-space modeling. They, too, have small standard deviations in both the  $T = 50$  and  $T = 300$  conditions.

Overall, there seems to be little difference between state-space modeling and classical P-technique modeling of highly auto- and lagged cross-correlated multivariate time-series in recovering the parameters and the latent state process associated with the underlying state-space model.

## **AN EMPIRICAL APPLICATION**

To illustrate empirically the ideas we have been examining, analyses were conducted on a set of P-technique data first published by Lebo and Nesselroade (1978).<sup>1</sup> The data consist of daily (103 days) self-reports on a set of affective adjective rating scales gleaned from the emotion/affect literature. The participant was a young married woman expecting her first child. The 27 adjective scales used in this analysis are listed as the variables in Table 3

**TABLE 3 P-technique Factor Loadings**

| Variable | Factor  |        |            |          |           |
|----------|---------|--------|------------|----------|-----------|
|          | Arousal | Energy | Well-Being | Vitality | Affection |
| Excited  | .89     | .00    | .00        | .00      | .00       |
| Aroused  | .75     | .00    | .00        | .00      | .00       |

|              |      |     |      |     |     |
|--------------|------|-----|------|-----|-----|
| Enthusiastic | .55  | .00 | .41  | .00 | .00 |
| Vigorous     | .00  | .00 | .00  | .92 | .00 |
| Energetic    | .00  | .96 | .00  | .00 | .00 |
| Active       | .00  | .90 | .00  | .00 | .00 |
| Peppy        | .00  | .00 | .00  | .84 | .00 |
| Lively       | .00  | .36 | .00  | .57 | .00 |
| Cheerful     | .00  | .00 | .96  | .00 | .00 |
| Happy        | .00  | .00 | .77  | .00 | .17 |
| Glad         | .15  | .00 | .57  | .00 | .29 |
| Comfortable  | .00  | .00 | .89  | .00 | .00 |
| Contented    | .00  | .24 | .71  | .00 | .00 |
| Pleasant     | .00  | .00 | .94  | .00 | .00 |
| Carefree     | .00  | .00 | .86  | .00 | .00 |
| Relaxed      | .00  | .00 | .82  | .00 | .00 |
| At Ease      | .00  | .00 | .90  | .00 | .00 |
| Calm         | -.39 | .00 | .70  | .00 | .00 |
| Forgiving    | .00  | .00 | .00  | .00 | .83 |
| Friendly     | .00  | .00 | .00  | .00 | .90 |
| Affectionate | .34  | .00 | .00  | .00 | .66 |
| Kindly       | .00  | .00 | .00  | .00 | .86 |
| Warmhearted  | .00  | .00 | .42  | .00 | .54 |
| Anxious      | .67  | .00 | -.63 | .00 | .00 |
| Cautious     | .00  | .00 | .00  | .00 | .00 |
| Tired        | .00  | .00 | -.41 | .00 | .00 |
| Efficient    | .00  | .00 | .78  | .00 | .00 |

Note. Expected salient loadings in boldface.

An orthogonal, exploratory, five-factor P-technique model was fitted to the (27,27)-dimensional correlation matrix by the method of maximum likelihood. The Standardized Root Mean Residual (RMR) = 0.027, the Non-Normed Fit Index (NNFI) = 0.96, and the Comparative Fit Index (CFI) = 0.97 (CFI is larger than 0.95, hence indicating excellent fit). These three goodness-of-fit indices have been shown to perform well in large-scale simulation studies (Brown, 2006).

This exploratory five-factor P-technique solution was then obliquely rotated to the oblimin criterion. Finally, all of the oblimin loadings smaller than 0.3 were fixed at zero and the confirmatory five-factor P-technique model thus defined was again fitted to the (27,27)-dimensional correlation matrix. This loading pattern along with the factor intercorrelations that are presented in Table 3 and Table 4, respectively, comprised the P-technique solution. The factor labels are interpretations based on the pattern of factor loadings. The three fit indices mentioned previously continued to indicate very good fit: SRMR = 0.047, NNFI = 0.95, and CFI = 0.95. This solution was used to estimate the five-dimensional latent factor series for comparison purposes described subsequently.

**TABLE 4 P-technique Factor Intercorrelations**

|                   | <b>Arousal</b> | <b>Energy</b> | <b>Well-Being</b> | <b>Vitality</b> | <b>Affection</b> |
|-------------------|----------------|---------------|-------------------|-----------------|------------------|
| <b>Arousal</b>    | 1.00           |               |                   |                 |                  |
| <b>Energy</b>     | .71            | 1.00          |                   |                 |                  |
| <b>Well-Being</b> | .43            | .70           | 1.00              |                 |                  |

|           |     |     |     |      |      |
|-----------|-----|-----|-----|------|------|
| Vitality  | .73 | .85 | .58 | 1.00 |      |
| Affection | .39 | .71 | .91 | .57  | 1.00 |

For comparison with the P-technique outcome, a confirmatory five-dimensional state-space model was fitted to the (54,54)-dimensional block Toeplitz matrix containing the 0-lag and 1-lag correlations. The pattern of factor loadings was the same as for the confirmatory five-factor P-technique model shown in Table 3. All nonsignificant entries in the (5,5)-dimensional matrix of auto- and cross-regression coefficients were fixed at zero. The results of fitting the five-dimensional state-space model are shown in two tables—the loadings are presented in Table Table 5 and the (5,5)-dimensional matrix of auto- and cross-regression coefficients is given in Table Table 6

**TABLE 5 State-Space Loadings**

| Variable     | Factor  |        |            |          |           |
|--------------|---------|--------|------------|----------|-----------|
|              | Arousal | Energy | Well-Being | Vitality | Affection |
| Excited      | m1.00   | .00    | .00        | .00      | .00       |
| Aroused      | .85     | .00    | .00        | .00      | .00       |
| Enthusiastic | .61     | .00    | .42        | .00      | .00       |
| Vigorous     | .00     | .00    | .00        | 1.00     | .00       |
| Energetic    | .00     | 1.00   | .00        | .00      | .00       |
| Active       | .00     | .94    | .00        | .00      | .00       |
| Peppy        | .00     | .00    | .00        | .91      | .00       |
| Lively       | .00     | .37    | .00        | .62      | .00       |
| Cheerful     | .00     | .00    | m1.00      | .00      | .00       |
| Happy        | .00     | .00    | .77        | .00      | .25       |
| Glad         | .17     | .00    | .56        | .00      | .39       |
| Comfortable  | .00     | .00    | .92        | .00      | .00       |
| Contented    | .00     | .25    | .74        | .00      | .00       |
| Pleasant     | .00     | .00    | .97        | .00      | .00       |
| Carefree     | .00     | .00    | .89        | .00      | .00       |
| Relaxed      | .00     | .00    | .85        | .00      | .00       |
| At Ease      | .00     | .00    | .93        | .00      | .00       |
| Calm         | -.44    | .00    | .73        | .00      | .00       |
| Forgiving    | .00     | .00    | .00        | .00      | 1.00      |
| Friendly     | .00     | .00    | .00        | .00      | 1.07      |
| Affectionate | .39     | .00    | .00        | .00      | .78       |
| Kindly       | .00     | .00    | .00        | .00      | 1.04      |
| Warmhearted  | .00     | .00    | .37        | .00      | .72       |
| Anxious      | .76     | .00    | -.66       | .00      | .00       |
| Cautious     | .00     | .00    | .00        | .00      | .00       |
| Tired        | .00     | .00    | -.43       | .00      | .00       |
| Efficient    | .00     | .00    | .82        | .00      | .00       |

Note. Expected salient loadings in boldface.

**TABLE 6 Regression Coefficients Linking the States at  $t$  and  $t + 1$  in the Latent VAR(1) Model for the Five-Variate State Process (—= Value Fixed at 0)**

|                    | Arousal(t) | Energy(t) | Well-Being(t) | Vitality(t) | Affection(t) |
|--------------------|------------|-----------|---------------|-------------|--------------|
| Arousal (t + 1)    | .25        | —         | —             | —           | —            |
| Energy (t + 1)     | —          | —         | .35           | —           | —            |
| Well-Being (t + 1) | -.37       | .21       | .46           | —           | —            |
| Vitality (t + 1)   | —          | .26       | —             | —           | —            |
| Affection (t + 1)  | -.21       | —         | —             | —           | .61          |

The three fit indices used earlier to examine the P-technique model fit are, for the state-space model, SRMR = 0.077, NNFI = 0.89, and CFI = 0.90. These values are not as impressive as they were for the confirmatory five-factor P-technique model. Yet all the modification indices in model matrices (LY, BE, and PS) are small so the model was accepted as is.

Using the parameter estimates of the confirmatory five-dimensional state-space model, the Kalman filtered estimates of the five-variate state process were determined and these were correlated with the corresponding five-variate latent factor series obtained by means of P-technique. This yielded the following values:

Arousal: +.99  
 Energy: +.72  
 Well-Being: +.92  
 Vitality: +.86  
 Affection: +.94

The lower intercorrelation of the two sets of estimates for Energy may be largely due to the fact that this factor series loads substantially on only two items (Energetic and Active). Similarly, the second lowest intercorrelation (+.86) for Vitality may be due to its large loadings on only two variables (Vigorous and Peppy). The remaining three factor series yield very high intercorrelations between the P-technique and state-space estimates.

## DISCUSSION AND CONCLUSIONS

Keeping in mind that our primary objective in this article is to evaluate the ability of the classical P-technique model to represent the factorial structure of a multivariate time-series as accurately and reliably as does state-space modeling, our choice to fit a model that follows closely from that of Molenaar (1985) was made for two primary reasons. First, it is a natural extension of P-technique. Second, recently available exact Maximum Likelihood estimation techniques (Hamaker, Dolan, & Molenaar, 2005) yield estimates for the state-space model that closely correspond to the estimates obtained by means of Molenaar's (1985) approach (Van Rijn, 2006). Note that the close correspondence is between the estimates themselves, not the estimated standard errors and chi-square goodness-of-fit values.

The correlations between estimated and true state processes are very impressive for both P-technique and state-space modeling. More to the point of this article, however, the results of the present simulation study show that P-technique yields valid parameter estimates. Indeed, P-technique in combination with the regression estimator for factor scores very reliably recovers the underlying latent factor series (state process). Its performance is comparable to that of the Kalman filter in combination with state-space modeling. As was pointed out by Molenaar (1985), however, in the case where latent factor series have lead-lag relations with manifest variable series, neither P-technique nor state-space modeling can be expected to provide valid estimates. In such cases one needs a full dynamic factor model with lagged loadings as presented, for example, in Molenaar (1985).

An implication of these admittedly limited findings is that a historically important and intuitively appealing procedure (P-technique factor analysis), the key aspects of which are easily communicated to others, can be used to secure a quick but apparently accurate approximation to the outcomes of applying more sophisticated (but also more difficult to program and interpret) methods for studying processes via multivariate time-series. The outcomes of our simulations and the empirical example suggest that much of what has already been established via traditional P-technique factor analysis, such as the

trait-state distinction, is still on a relatively solid footing even as newer methods for modeling multivariate time-series are becoming more and more attractive to researchers.

## ACKNOWLEDGMENT

This work was supported by the Institute for Developmental and Health Research Methodology at the University of Virginia.

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## Notes

<sup>1</sup> We are grateful to Michael A. Lebo for permission to use these data.

## List of Tables

**TABLE 1 True Factor Loadings and Those Estimated via State-Space and Conventional P-technique Modeling (Standard Deviations in Parentheses)**

| Variable | Estimated Loading |          |                   |             |                   |             |
|----------|-------------------|----------|-------------------|-------------|-------------------|-------------|
|          | True Loading      |          | State-Space Model |             | P-technique Model |             |
|          | Factor 1          | Factor 2 | Factor 1          | Factor 2    | Factor 1          | Factor 2    |
| 1        | 1.000             | .000     | 1.000             | .000        | 1.000             | .000        |
| 2        | .900              | .700     | .919 (.211)       | .697 (.222) | .918 (.235)       | .695 (.242) |
| 3        | .800              | .800     | .823 (.191)       | .793 (.216) | .827 (.249)       | .784 (.319) |
| 4        | .700              | .900     | .710 (.198)       | .898 (.229) | .708 (.228)       | .899 (.267) |
| 5        | .000              | 1.000    | .000              | 1.000       | .000              | 1.000       |
| 1        | 1.000             | .000     | 1.000             | .000        | 1.000             | .000        |
| 2        | .900              | .700     | .906 (.065)       | .697 (.071) | .909 (.119)       | .694 (.133) |
| 3        | .800              | .800     | .804 (.066)       | .798 (.074) | .808 (.118)       | .793 (.137) |
| 4        | .700              | .900     | .704 (.067)       | .894 (.078) | .710 (.120)       | .888 (.144) |
| 5        | .000              | 1.000    | .000              | 1.000       | .000              | 1.000       |

Note. All estimates are based on 500 replications. Time-series are 50 occasions in length (upper portion of table) and 300 occasions in length (lower portion of table).

**TABLE 2 Averaged Correlations Between True Process  $\eta_t$  and Estimates of  $\hat{\eta}_t$ . Averages Are Over 500 Replications (Standard Deviations are in Parentheses)**

| Variables Correlated   | Factor 1    | Factor 2    |
|--|-------------|-------------|
| $\bar{r}$ [true $\eta_t$ - Kalman filtered $\hat{\eta}_t$ ]      | .933 (.035) | .917 (.046) |
| $\bar{r}$ [true $\eta_t$ - regression estimated $\hat{\eta}_t$ ] | .923 (.035) | .906 (.040) |
| $\bar{r}$ [true $\eta_t$ - Kalman filtered $\hat{\eta}_t$ ]      | .957 (.009) | .944 (.009) |
| $\bar{r}$ [true $\eta_t$ - regression estimated $\hat{\eta}_t$ ] | .949 (.014) | .934 (.016) |

Note. Time-Series are 50 Occasions in Length (Upper Portion of Table) and 300 Occasions in Length (Lower Portion of Table).

**TABLE 3 P-technique Factor Loadings**

| Variable     | Factor  |        |            |          |           |
|--------------|---------|--------|------------|----------|-----------|
|              | Arousal | Energy | Well-Being | Vitality | Affection |
| Excited      | .89     | .00    | .00        | .00      | .00       |
| Aroused      | .75     | .00    | .00        | .00      | .00       |
| Enthusiastic | .55     | .00    | .41        | .00      | .00       |
| Vigorous     | .00     | .00    | .00        | .92      | .00       |
| Energetic    | .00     | .96    | .00        | .00      | .00       |
| Active       | .00     | .90    | .00        | .00      | .00       |
| Peppy        | .00     | .00    | .00        | .84      | .00       |
| Lively       | .00     | .36    | .00        | .57      | .00       |
| Cheerful     | .00     | .00    | .96        | .00      | .00       |
| Happy        | .00     | .00    | .77        | .00      | .17       |
| Glad         | .15     | .00    | .57        | .00      | .29       |
| Comfortable  | .00     | .00    | .89        | .00      | .00       |
| Contented    | .00     | .24    | .71        | .00      | .00       |
| Pleasant     | .00     | .00    | .94        | .00      | .00       |
| Carefree     | .00     | .00    | .86        | .00      | .00       |
| Relaxed      | .00     | .00    | .82        | .00      | .00       |
| At Ease      | .00     | .00    | .90        | .00      | .00       |
| Calm         | -.39    | .00    | .70        | .00      | .00       |
| Forgiving    | .00     | .00    | .00        | .00      | .83       |
| Friendly     | .00     | .00    | .00        | .00      | .90       |
| Affectionate | .34     | .00    | .00        | .00      | .66       |
| Kindly       | .00     | .00    | .00        | .00      | .86       |
| Warmhearted  | .00     | .00    | .42        | .00      | .54       |
| Anxious      | .67     | .00    | -.63       | .00      | .00       |
| Cautious     | .00     | .00    | .00        | .00      | .00       |
| Tired        | .00     | .00    | -.41       | .00      | .00       |
| Efficient    | .00     | .00    | .78        | .00      | .00       |

Note. Expected salient loadings in boldface.

**TABLE 4 P-technique Factor Intercorrelations**

| Arousal | Energy | Well-Being | Vitality | Affection |
|---------|--------|------------|----------|-----------|
|---------|--------|------------|----------|-----------|

|            |      |      |      |      |      |
|------------|------|------|------|------|------|
| Arousal    | 1.00 |      |      |      |      |
| Energy     | .71  | 1.00 |      |      |      |
| Well-Being | .43  | .70  | 1.00 |      |      |
| Vitality   | .73  | .85  | .58  | 1.00 |      |
| Affection  | .39  | .71  | .91  | .57  | 1.00 |

TABLE 5 State-Space Loadings

| Variable     | Factor       |        |              |          |           |
|--------------|--------------|--------|--------------|----------|-----------|
|              | Arousal      | Energy | Well-Being   | Vitality | Affection |
| Excited      | <b>m1.00</b> | .00    | .00          | .00      | .00       |
| Aroused      | .85          | .00    | .00          | .00      | .00       |
| Enthusiastic | .61          | .00    | .42          | .00      | .00       |
| Vigorous     | .00          | .00    | .00          | 1.00     | .00       |
| Energetic    | .00          | 1.00   | .00          | .00      | .00       |
| Active       | .00          | .94    | .00          | .00      | .00       |
| Peppy        | .00          | .00    | .00          | .91      | .00       |
| Lively       | .00          | .37    | .00          | .62      | .00       |
| Cheerful     | .00          | .00    | <b>m1.00</b> | .00      | .00       |
| Happy        | .00          | .00    | .77          | .00      | .25       |
| Glad         | .17          | .00    | .56          | .00      | .39       |
| Comfortable  | .00          | .00    | .92          | .00      | .00       |
| Contented    | .00          | .25    | .74          | .00      | .00       |
| Pleasant     | .00          | .00    | .97          | .00      | .00       |
| Carefree     | .00          | .00    | .89          | .00      | .00       |
| Relaxed      | .00          | .00    | .85          | .00      | .00       |
| At Ease      | .00          | .00    | .93          | .00      | .00       |
| Calm         | -.44         | .00    | .73          | .00      | .00       |
| Forgiving    | .00          | .00    | .00          | .00      | 1.00      |
| Friendly     | .00          | .00    | .00          | .00      | 1.07      |
| Affectionate | .39          | .00    | .00          | .00      | .78       |
| Kindly       | .00          | .00    | .00          | .00      | 1.04      |
| Warmhearted  | .00          | .00    | .37          | .00      | .72       |
| Anxious      | .76          | .00    | -.66         | .00      | .00       |
| Cautious     | .00          | .00    | .00          | .00      | .00       |
| Tired        | .00          | .00    | -.43         | .00      | .00       |
| Efficient    | .00          | .00    | .82          | .00      | .00       |

Note. Expected salient loadings in boldface.

TABLE 6 Regression Coefficients Linking the States at  $t$  and  $t + 1$  in the Latent VAR(1) Model for the Five-Variate State Process (—= Value Fixed at 0)

|                           | <b>Arousal(t)</b> | <b>Energy(t)</b> | <b>Well-Being(t)</b> | <b>Vitality(t)</b> | <b>Affection(t)</b> |
|---------------------------|-------------------|------------------|----------------------|--------------------|---------------------|
| <b>Arousal (t + 1)</b>    | .25               | —                | —                    | —                  | —                   |
| <b>Energy (t + 1)</b>     | —                 | —                | .35                  | —                  | —                   |
| <b>Well-Being (t + 1)</b> | -.37              | .21              | .46                  | —                  | —                   |
| <b>Vitality (t + 1)</b>   | —                 | .26              | —                    | —                  | —                   |
| <b>Affection (t + 1)</b>  | -.21              | —                | —                    | —                  | .61                 |